Discussion Comment: Entropy in Cosmology

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In the first few years as I began my Ph.D. studies in 1954 under John A. Wheeler's direction he stated quite explicitly one of his strategies. I believe he called it ``dynamic conservatism", although I may have the adjective wrong. But the idea was clear: take some of the best grounded theories available and push their solutions far beyond the known experimental or observational realm. Explore what they have to tell us that we haven't yet seen, even in the mind's eye. Although his faith in quantum mechanics was solid, he found it more difficult to extrapolate there [but see his later 1967 work inspiring Bryce DeWitt, Phys.Rev.vol.160, p.1113] than with the classical theories of gravity and electromagnetism. With these two he and his students elaborated ``geons" and ``wormholes", both to good effect. The geon work led to a more solid understanding of gravitational waves (whose existence, even in theory, was then considered insecure) by showing in the gravitational geon and in the Brill wave initial conditions that bundles of such waves could produce Newtonian type centers of gravitational attraction, hence effectively had positive energy. The wormhole work, by demanding through its nontrivial topology a rigorously coordinate independent view of spacetime, advertised the tools that would allow a clear understanding of the nature of classical black holes.

How does this tradition bear on entropy? The conservative aspect is what I want to emphasize. Wheeler used theories that had clear and unambiguous statements. When entropy is used in the cosmological context I, like many others, feel the concept needs clarification. One tool that helps enforce conservatism and lead to clarity is the demand for correspondence principles. Anything newly proposed should at least be able to give the known answers in familiar domains. Thus a search for good entropy definitions in cosmology should begin with a thorough understanding of entropy in Newtonian gravity. Here my guide would be Lynden-Bell [cond-mat/9812172] where he summarizes the studies of self-gravitating systems such as isothermal gasses (whose molecules may be stars in a cluster). Such systems, as astronomers have long known, may have negative heat capacities. One consequence of this that Lynden-Bell expounds is that such systems in thermal contact do not attain thermal equilibrium. Thus the heat capacity C is not an extensive quantity, and the specific heat cannot be defined. Since thermodynamic entropy is computed as the integral of (C/T)dT we can expect peculiarities in the entropy to appear also. One suspects that adding up the locally measured entropy densities of matter in the universe does not give the correct total entropy if that could be defined.

For cosmology the negative heat capacities of gravitational systems are of paramount importance for the evolution of structure in the universe. This point has been made independently by Fang Li Xhi [see his Chapter 'How Order was Born of Chaos' in the book ``Creation of the Universe" by Fang and Li, English translation 1989, World Scientific Publishing Co., ISBN 9971-50-601-7] and by Freeman Dyson [unpublished lecture 'Life in the Universe ...', Univ. of Maryland, 3

December 1999]. It is also the comment that gravitational systems are unstable and do not attain thermodynamic equilibrium.

In the present state of our knowledge, I do not believe that the entropy of the universe has a useful definition, and I am not convinced that a good definition must be consistent with the assertion that the entropy can be expected to increase as the universe evolves.

But let us explore some gravitational systems where a straightforward application of thermodynamics seems possible. As a first case consider a gas cloud that is preparing to form itself into a star or solar system.

If the system has evolved enough to have a large density contrast from center to edge (as in Lynden-Bell's examples), it will have a negative heat capacity. To calculate entropy thermodynamically one needs to connect two equilibrium thermodynamic states by a reversible process.

Thus I want the gas cloud to be placed in a (very expensive) oven that has the necessary feedback and control to maintain stability. Thus it will adjust its temperature to be just above or just below that of the gas (assuming the two interact primarily via infrared radiation, as the external pressure for the gas can be negligible) depending on whether we wish to heat the gas reversibly or to cool it. The natural evolution of the gas cloud would be to contract and heat up. To progress through these evolutionary steps reversibly, the oven must keep itself slightly cooler than the gas so that the gas cloud will reversibly radiate energy to the oven and thus heat up according to the virial theorem. The oven's feedback will control the rate by heating itself a bit as needed to cool the gas should the gas be heating more rapidly than desired. As the gas is thus led through a reversible thermodynamic process, its entropy change can be validly calculated by $dU = T\,dS$ (assuming a fixed volume oven or zero external pressure). Here dU is negative, being the energy change of the gas arising from the infrared radiation it is allowed to send to the oven. In this way we conclude that a hot dense gas cloud has lower entropy than a cooler more diffuse cloud containing the same number of baryons but a milder gravitational field. I cannot see that integrating the specific entropy of local regions of the cloud (multiplied by the density) over the entire cloud and ignoring the gravitational field would give the same quantitative result.

The thermodynamic viewpoint also makes the entropy of a black hole not seem unduly large. Using the same astrophysical oven as before we can reversibly build a large black hole from a small one. The small black hole will have negligible entropy. (E.g., excite one Planck length mode of the electromagnetic, gravitational, or grand unified field to a one quantum state which may be presumed to give a Planck size black hole and have entropy zero.) By programming the oven to stay just slightly hotter than the black hole, it will dump radiation into the black hole, raising its mass and reducing its Hawking temperature. In the usual $dS = Dq/T$ thermodynamic formula for entropy change in a reversible process we set the heat input to be the mass increase $dQ = dM$ and use the Hawking temperature $T = 1/(8 \pi M)$ to find by integration the usual Hawking entropy. We thus get an idea of the amount of information consumed in building a solar mass black hole reversibly: Near solar mass ($Mc^2 = 10^{66}$ eV) size the black hole must be fed with Hawking temperature (5×10^{-12} eV or 65 nanoKelvin) quanta, so adding a solar mass requires about 2 x 10^77 quanta, the same number as the Hawking entropy, 10^77 bits, for a solar mass black hole. Core collapse in a supernova is very irreversible, but with the same endpoint black hole, it must achieve the same final entropy.